NO, E DOES NOT EQUAL MC^2 AND I'LL SHOW YOU WHY!

Dedication

To the memory of Stephen Hawking

January 8^{th}, 1942

to

March 14, 2018

Perhaps the universe itself lauded him.

He was born on the 300^{th} anniversary of Galileo's death,

and he died on π day (3 / 14),

which was also Einstein's birthday!

Questions

I’m quite open to questions during the presentation. But…

- We’ve got a *lot* to get through, and a limited time to do so in, so if things even *start* to get out of hand, I may have to be firm and cut off the discussion.
- There are almost 60 slides, which allows me about 2 minutes per slide, even without questions. Time will be tight.
- Again, for time constraint reasons, I’ll accept only questions that boil down to “I don’t understand”, “could you go over that again” and so on. If any other types of questions are asked, I’ll have to be curt and say that we have to defer the question.
- When the presentation is over, if we have any time left, then the field will be open to any questions.

Confession Time

- This talk really should have been titled
    - A Technical Introduction to Special Relativity
- But with a title like *No, E Does NOT Equal mc^2 and I'll Show You Why!*, well, if that didn't convince you to come to this presentation, nothing would! ☺
- Hawking, in his introduction to *A Brief History of Time*, said that he was advised that every equation in his book would cut his sales in half. He hoped that $E=mc^2$ could sneak by.
- But without the equations you can't *really* understand Relativity. So it's my goal to descend from a 10,000 meter overview of the subject into something that can really describe what's going on.

My Credentials

My Credentials

In other words, I have none.

I’m not a physicist. I’m just a computer nerd who likes math and fundamental physics.

So while I believe that what I’m about to talk about is true, you should probably take it *cum grano salis*.

Bibliography and Links

- I used to put this at the end of the presentation, but after the last slide, I never got to present these. So here they are at the front.
- Albert Einstein
  - On the Electrodynamics of Moving Bodies (June 30, 1905) – http://www.fourmilab.ch/etexts/einstein/specrel/www/
  - Does the Inertia of a Body Depend upon its Energy-Content? (September 27, 1905) – http://users.physik.fu-berlin.de/~kleinert/files/e_mc2.pdf
- Special Relativity in Wikipedia
  - https://en.wikipedia.org/wiki/Special_relativity
  - https://en.wikipedia.org/wiki/Special_relativity#Time_dilation
  - https://en.wikipedia.org/wiki/Special_relativity#Length_contraction
  - https://en.wikipedia.org/wiki/Special_relativity#Composition_of_velocities
  - https://en.wikipedia.org/wiki/Relativistic_Doppler_effect
  - https://en.wikipedia.org/wiki/Planck%E2%80%93Einstein_relation

Bibliography and Links

- Mr. Tompkins books by George Gamow
  - https://www.amazon.com/s/ref=nb_sb_noss_2?url=search-alias%3Daps&field-keywords=Mr.+Tompkins&rh=i%3Aaps%2Ck%3AMr.+Tompkins
- Associated videos
  - Feynman – How did we find the speed of light – https://www.youtube.com/watch?v=b9F8Wn4vf5Y
  - Sagan – Speed of light constant – https://www.youtube.com/watch?v=_pEiA0-r5A8
- Special Relativity and Classical Field Theory – https://www.amazon.com/Special-Relativity-Classical-Field-Theory/dp/0465093345/ref=sr_1_3?ie=UTF8&qid=1521837735&sr=8-3&keywords=leonard+susskind
  - Other excellent Susskind books – https://www.amazon.com/s/ref=nb_sb_noss_2?url=search-alias%3Daps&field-keywords=leonard+susskind&rh=i%3Aaps%2Ck%3Aleonard+susskind

Special vs. General Relativity

- When I was young, I thought that the “General” in General Relativity referred to something rather vague, just sort of hand-wavy (wibbly-wobbly, timey-wimey?), as in “generally speaking”. Whereas Special Relativity was, oh boy, something really special.
- WRONG!
- Special Relativity was created first because Einstein couldn’t analyze motion in general. But he could understand simpler situations, involving the special case of particles that do not change speed or direction. These are called *inertial frames* where bodies (in the absence of other forces like friction) just continue to coast along through intertia, same speed, same direction.
- General Relativity is a much more sophisticated theory, allowing us to analyze particles that can slow down, can speed up, can dart off in arbitrary directions, and in general constantly curve.
- In this presentation, the term “Relativity” by itself always means “Special Relativity”.

Where We're Going

- We've all heard that under Special Relativity time and space are not what we normally expect. (We'll go into that later.)
- So after a number of preliminaries, we're going to consider a particle traveling inertially from two points of view.
  - Alice is traveling along with the particle, so that to her it seems at rest.
  - Bob is viewing it while traveling at some velocity v. But he can view himself at rest with our particle travelling at velocity v in the other direction.
  - The kinetic energy (energy of motion) of the particle will be zero in the first case, but non-zero in the second case.
- To reconcile these two views, we'll see that there's an interplay between energy and mass.

The Assumptions of Special Relativity

- The laws of nature are the same for all observers in uniform motion (i.e. constant speed and direction – this includes being at rest) relative to one another. (Galileo knew this.)
- The speed of light is always measured to be the same for all observers, regardless of their relative motion or the motion of the light source.
  - "The principle of the constancy of the velocity of light is of course contained in Maxwell's equations [for electromagnetism]" – Albert Einstein, *Does the Inertia of a Body Depend on its Energy-Content?*, September 27, 1905.
  - <u>*Very*</u> loosely speaking, Maxwell's equations are formulated in terms of electric/magnetic fields and *how they change* (differential equations, for those who know calculus). This has the subtle side effect of requiring that the speed of electromagnetic energy (i.e. light) must always be measured to be the same.

Batter Up!

- Using the examples from 100+ years ago, suppose someone is on a train moving at 100 km/hr, and can throw a baseball at 80 km/hr.
- You’re at rest on the ground and see the baseball thrown towards the front of the train. What speed would you measure the baseball to be travelling?
- 180 hm/hr, right? (Actually, no, but why is on different slide.)
- Now imagine she shines a flashlight towards the front of the train. Since realistic speeds are peanuts compared to the speed of light, let’s imagine the train moving at 100 km/second, and the light from the flashlight is, of course, moving at the speed of light, about 300,000 km/sec.
- You’d measure the light to be moving at 300,100 km/sec, right?
- Wrong! Maxwell’s equations say that you always measure the speed of light to be the same. We’ll see another reason why later.

Mathematical Preliminaries

- Later on we'll need to talk about $\gamma = 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + \frac{63}{256}\beta^{16} + \ \ldots$
- To make sure we're all up to speed some of the math involved, we're going to take a few minutes to build up the background we'll need. Once we're all comfortable with these, we can start to dig into the issues.
  - I'll assume everyone knows the Pythagorean Theorem
  - Exponents – We'll need to consider what it means to raise an expression to the power of -½., so we'll see how this can make sense.
  - The Binomial Theorem – It can be shown that raising an expression to the -½th power can be expressed as an infinite sum.
  - Summing an infinite series – But you'll need to understand that adding an infinite number of values can sum to a finite value.
  - Approximating an infinite sum. And we'll be able to approximate the finite value (in this case) by using only the first two of the infinite number of terms.

Exponents

- We all know that, say, $x^2 * x^3 = x^{2+3} = x^5$ (where * represents multiplication) and in general $x^a * x^b = x^{a+b}$. **We add the exponents.**

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- If (say), $x^0 * x^5 = x^{0+5} = x^5$, then x^0 must equal 1 (for all x).
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  - $(1 - x^2)^{-½} = \frac{1}{\sqrt{1 - x^2}}$

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    - $(1 - x^2)^{-½} = \dfrac{1}{\sqrt{1-x^2}}$
- Total aside: Can also be extended to irrational numbers (e.g. $x^{\sqrt{2}}$), and even imaginary numbers (involving *i*, the square root of -1)!
    - **$e^{i\pi} + 1 = 0$** Which many consider to be the most beautiful equation in all of mathematics

The Binomial Theorem

- Suppose you've got two values (say, x and y) and you want to raise their sum to a power
- For example
  - $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
  - $(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
  - $(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$
- There's a formula called the Binomial Theorem (https://en.wikipedia.org/wiki/Binomial_theorem) that can tell you how to expand the expression for any nonnegative integral exponent.
- A generalization of this allows any exponent (we'll need an exponent of -½), although in that case, you'll wind up with an infinite series. We'll need this shortly.

Summing an Infinite Series

- Suppose I give you a finite list of numbers, all greater than zero, and ask you to add them up and tell me the sum. No problem, right?
  - Bad little Tommy to teacher: "You told me my punishment was to add up this big column of numbers 10 times. I did so. And here are the 10 answers!"
- But suppose I give you an *infinite* list of such numbers. Well, you add an infinite number of positive numbers together and the answer will be infinite, right? WRONG!
- Now you have to *define* exactly what you mean by the sum of an infinite series, and there are several ways to do that. But there's an intuitive image…

Summing an Infinite Series

- Suppose you're at the far left of an 1 meter wide hallway. In half a second you advance half the distance to the far wall (we'll ignore atoms and quantum effects and assume that we can walk arbitrarily small distances in arbitrarily short times).
- After ½ second, you've walked one half the distance.
- After ¼ second you walk half the remaining distance – one quarter of the width.
- And so on.
- You'll never reach the far wall in a finite number of steps, but after an infinite number of steps you'll finally reach it. And you'll have walked those infinite number of steps in a single second. What we've shown is that…
- $1/2 + 1/4 + 1/8 + 1/16 + \ldots = 1$
- But really all we're establishing here is that adding an infinite series can give a finite result.
- http://platonicrealms.com/encyclopedia/Zenos-Paradox-of-the-Tortoise-and-Achilles
- https://en.wikipedia.org/wiki/Series_%28mathematics%29
- https://www.youtube.com/watch?v=prm5JHbMdRE

Summing an Infinite Series
A Bit More Mathematically

- Suppose we call the infinite summation *s* (for sum)
- s =   1/2 + 1/4 + 1/8 + 1/16 + …
- 2s = 1 + 1/2 + 1/4 + 1/8 + 1/16 + …
- 2s = 1 + s
- s = 1

Vocabulary:
Beta (β) and Especially Gamma (γ)

- Later we'll need the ratio of a velocity *v* compared to the speed of light *c* (from the Latin *celeritas* – speed).
- Physicists call this β (beta) = $\frac{v}{c}$.
- Which is turn leads to one of the most common and important calculations in Special Relativity called γ (gamma), which is
- $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = (1 - \frac{v^2}{c^2})^{-½}$
- Note that if v < c (i.e. slower than the speed of light) then the denominator is less than one, so its reciprocal is > 1.
  - $\gamma > 1$

γ – The Lorentz Transformation

- $\gamma = \dfrac{1}{\sqrt{1-\beta^2}} = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}} = (1 - \frac{v^2}{c^2})^{-½}$
- This is called the Lorentz Transformation.
- As we'll see later, this is the factor that will let us transform Alice's time into Bob's time, Alice's lengths into Bob's lengths, and so on. And vice versa.
- Using the Binomial Theorem, we can write this as an infinite series…
- $\gamma = 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + \frac{63}{256}\beta^{16} + \ \ldots$
- https://en.wikipedia.org/wiki/Lorentz_transformation

Approximations

- The speed of light (in a vacuum) is c = 299,792,458 meters / second.
  - Side note: This is exact. These days a meter is no longer defined as the length of a metal bar in Paris. A meter is *defined* as 1 / 299,792,458 of the speed of light.
- An average speed for a rifle bullet is 1000 m/s. Pretty fast by normal standards, but compared to the speed of light it's puny.
- In this case β = 1000 / 299,792,458 = 0.0000033356409519815 or 3 millionths of the speed of light.
- And γ is worse. It's 1.0000000000055633.
- So those are the exact values (well, exact out to umpteen decimal places). But let's see what happens when we use the Binomial expansion.

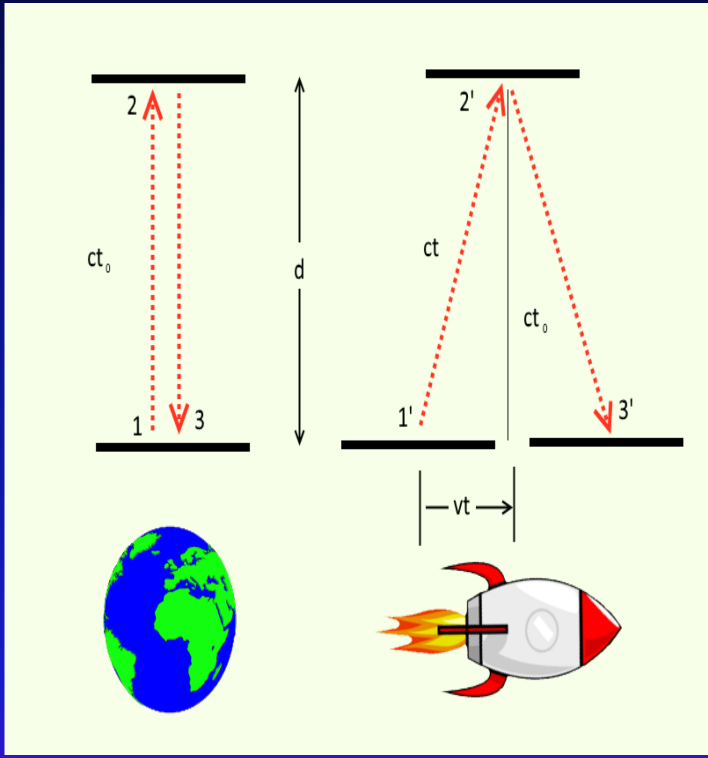
The Binomial Approximation of γ

- As we've seen, the value of γ can be calculated by an infinite series. But the table below shows that even at the speed of 1,000,000 m/sec, it's extremely accurate to calculate it by using only 2 terms of the series: $1 + \beta^2 / 2$. So we'll use that later.
  - Einstein did this. In his 1905 paper *Does the Inertia of a Body Depend on Its Energy Content?*, he said "Neglecting magnitudes of 4th and higher orders…"

Speed (m/sec)	γ (to 10 decimal places)	# of Terms	Value from series	% Difference
1,000	1.0000000000	2	1.0000000000	0.00000%
		3	1.0000000000	0.00000%
		4	1.0000000000	0.00000%
1,000,000	1.0000055633	2	1.0000055633	0.00000%
		3	1.0000055633	0.00000%
		4	1.0000055633	0.00000%

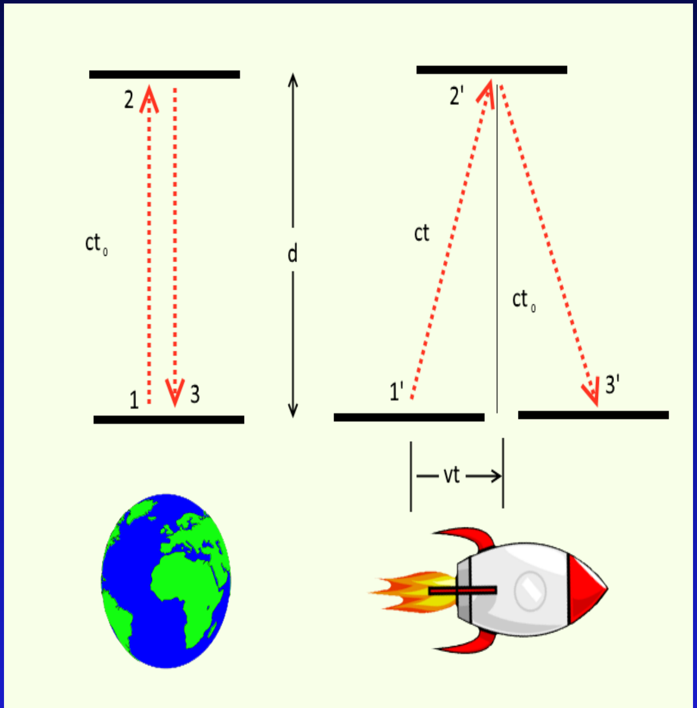
Time Dilation

- Imagine the following scenario:
- Alice is sitting home on Earth at rest.
- Through her telescope she sees Bob's rocket ship moving left to right at positive velocity v.
- They both have special clocks that are merely two mirrors, one above the other, with a photon bouncing between them. The two clocks are identical with the same height.
- Both Alice and Bob see their respective photons bouncing straight up and down. But Alice sees Bob's photon tracing a zig-zag path, each leg being longer than hers.
- In the time it takes Alice's clock to tick once, Bob's clock hasn't had a chance to do so yet.
- Bob's clock is running slower than Alice's!



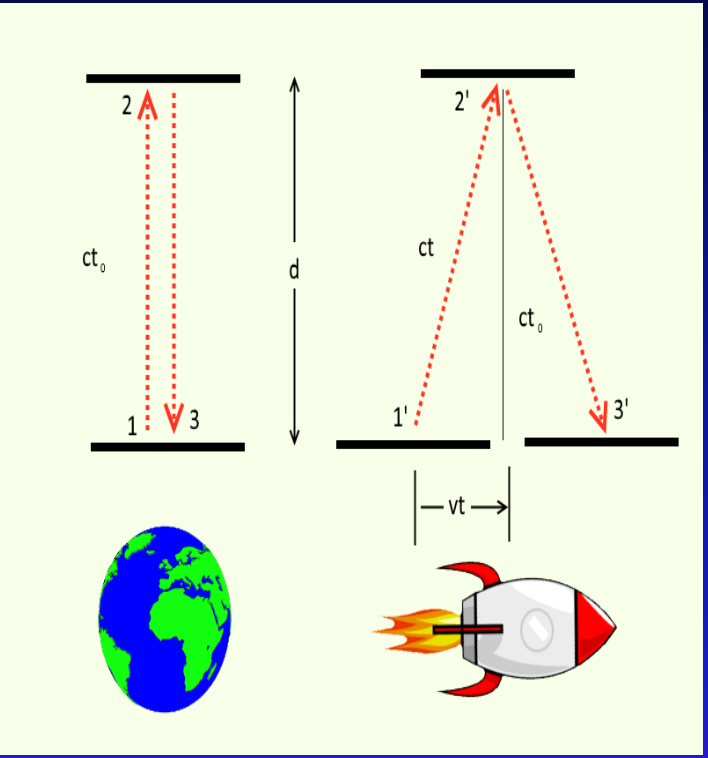
What Are the Pieces of the Puzzle?

- It takes Alice's clock some amount of time to go from one mirror to the other. We'll call this t_{Alice}. Note that in the diagram this is called t_0.
- Photons always travel at speed c, so the distance between the plates is ct_{Alice}.
- Because Bob's ship is moving horizontally, vertical distances are not affected, so the distance his photon travels is also ct_{Alice}.
- But note that the path of his photon from Alice's point of view is clearly longer than Alice's, so it will take longer to reach the top mirror (again, from Alice's POV). We'll call this time t_{Bob} (which in the diagram it's called just t). The distance is ct_{Bob}.
- Finally, in the time for it to reach the top, Bob's ship has traveled vt_{Bob}.



How Much Slower is Bob's Clock Running?

- So what's the relationship between t_{Alice} and t_{Bob}? We'll use the Pythagorean Theorem.
- It's clear that $(ct_{Bob})^2 = (ct_{Alice})^2 + (vt_{Bob})^2$.



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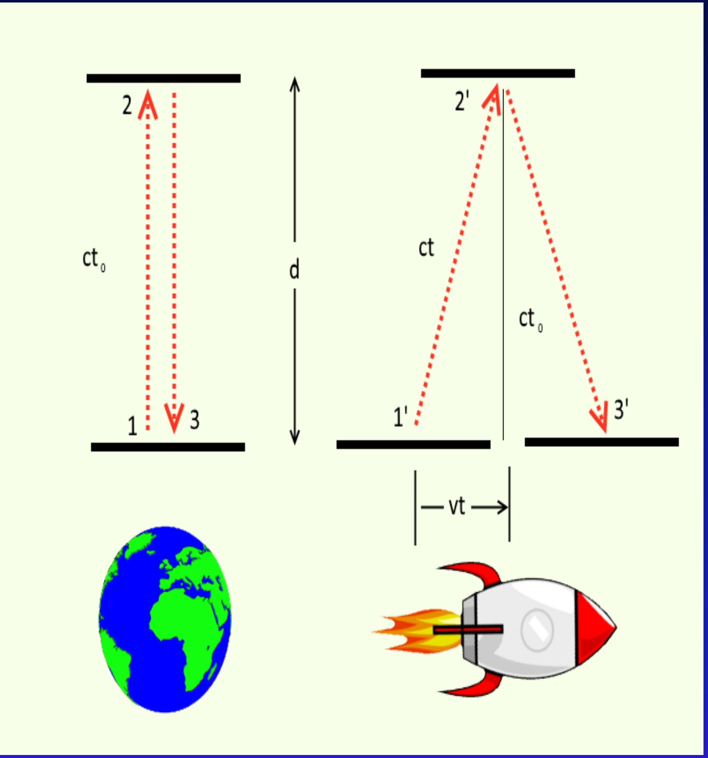

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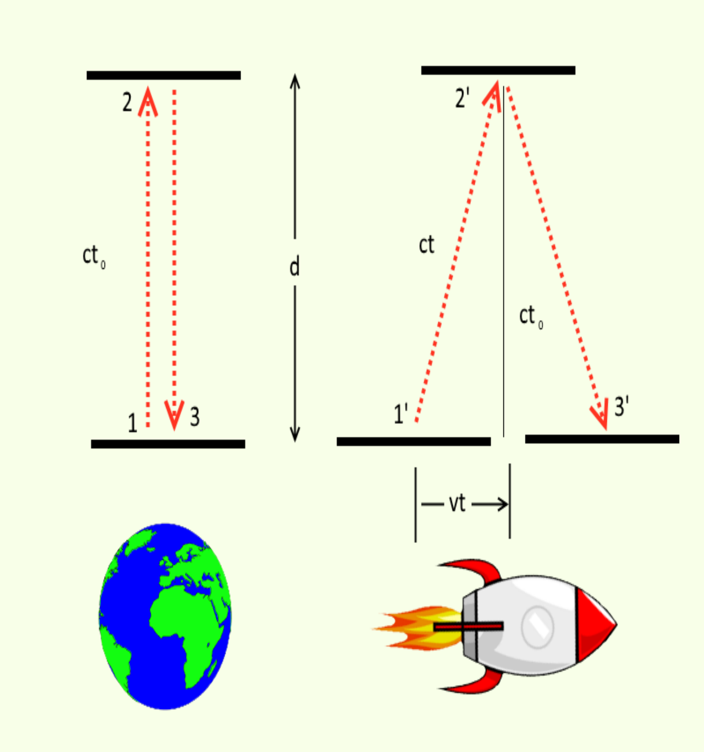
- $c^2 t_{Bob}^{\;2} = c^2 t_{Alice}^{\;2} + v^2 t_{Bob}^{\;2}$
- Divide both sides by c^2: $t_{Bob}^{\;2} = t_{Alice}^{\;2} + t_{Bob}^{\;2} \frac{v^2}{c^2}$



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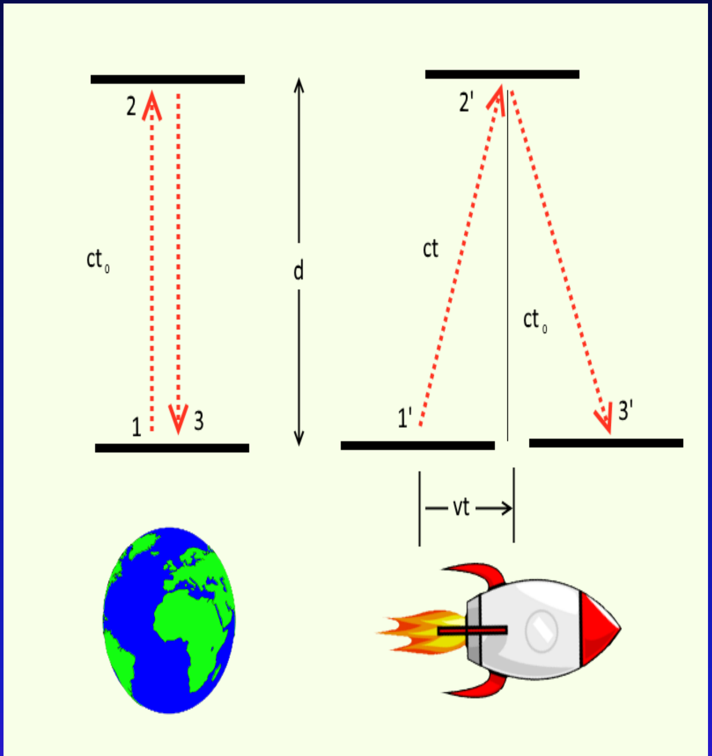
- Divide both sides by c^2: $t_{Bob}{}^2 = t_{Alice}{}^2 + t_{Bob}{}^2 \frac{v^2}{c^2}$
- So $t_{Bob}{}^2(1 - \frac{v^2}{c^2}) = t_{Alice}{}^2$



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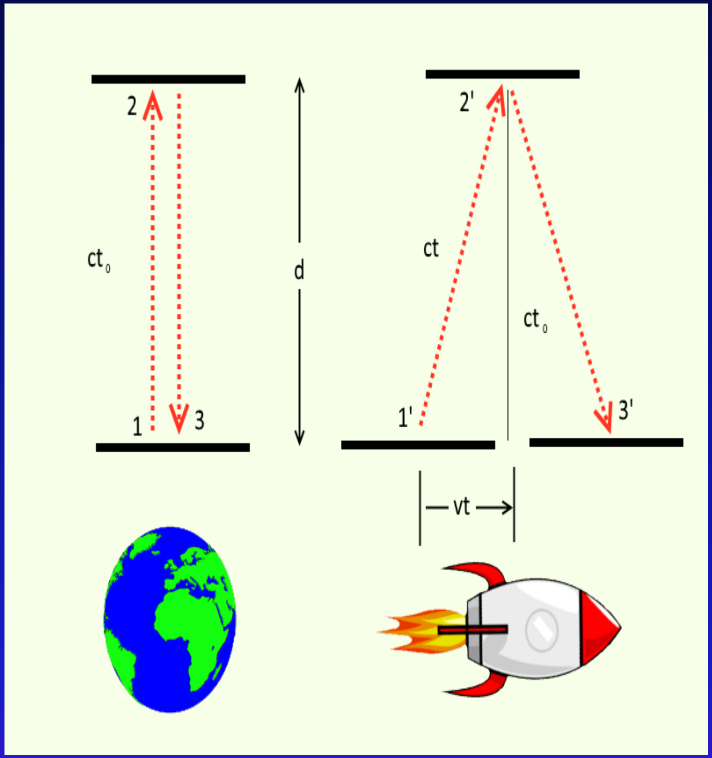
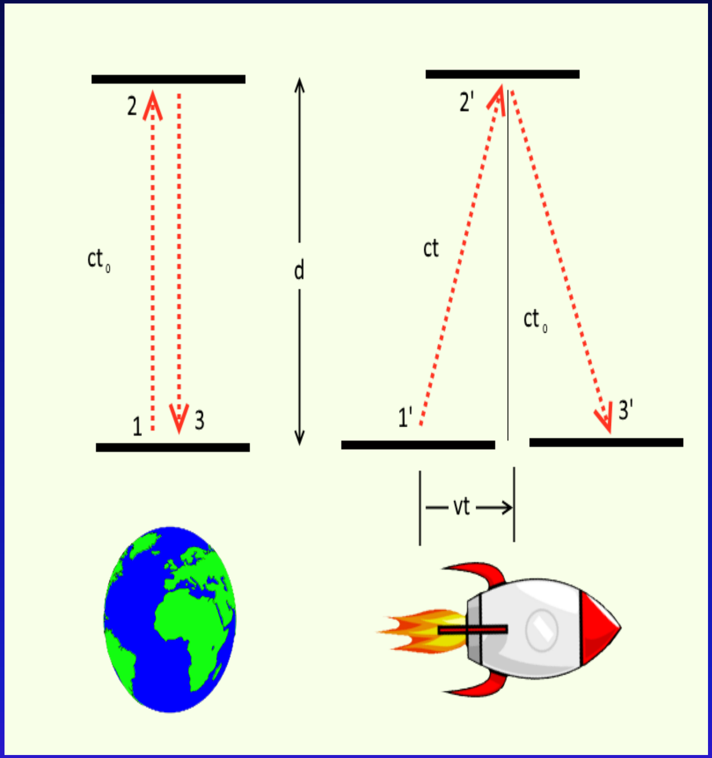
- Divide both sides by c^2: $t_{Bob}{}^2 = t_{Alice}{}^2 + t_{Bob}{}^2 \frac{v^2}{c^2}$
- So $t_{Bob}{}^2 (1 - \frac{v^2}{c^2}) = t_{Alice}{}^2$
- Dividing and taking the square root we have finally
- $t_{Bob} = t_{Alice} \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right) = t_{Alice}\, \gamma$
- So to get Bob's time (in his moving ship), we multiply Alice's (at rest) time by γ.
- And to convert Bob's time to Alice's, we'd just divide by γ.

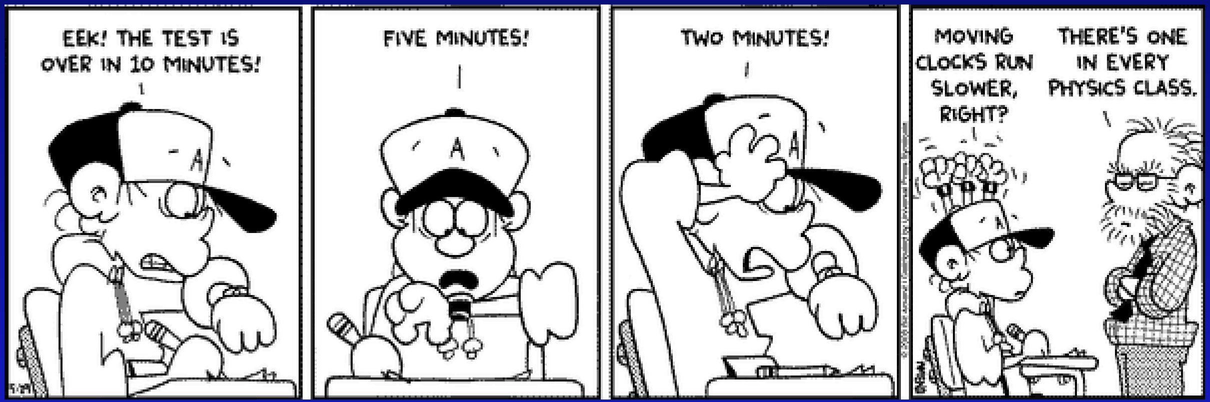


The Bottom Line

- Again: $t_{Bob} = t_{Alice} \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right) = t_{Alice}\,\gamma$
- Or: $t_{Alice} = t_{Bob} / \gamma$
- What is this telling us? Note that v is always less than c (you can't travel faster than light (Star Trek et al notwithstanding)).
- So if γ is, say, 2 (which happens when v is about 86.6% the speed of light), then $t_{Bob} = 2 * t_{Alice}$
- So for every tick of Bob's clock, Alice's ticks twice!
- Bob's clock ticks twice as slowly as Alice's does.
- But he doesn't notice anything different! His clock's traveling at speed v, but so is he!



Whew! Let's Give Our Brains a Break



An Act of Desperation

- In 1900, Max Planck was faced with a dilemma. There were two formulas that predicted what color light (i.e. frequency) an object (e.g. a piece of metal) emitted when heated. But neither was right. One worked well for low frequencies but not high ones, and the other was good for high frequencies only.
- He tried everything he could think of, but couldn't deduce what the correct formula was. Finally, in what he termed an "act of desperation", he assumed that energy could *not* take on all continuous values, but could only exist in discrete steps. This led to the correct formula for all frequencies.
- Thus was born Quantum Mechanics.
- The energy of light of frequency f is hf where h is Planck's constant.
  - Note: Physicists often use the Greek letter ν (nu) for frequency, so you might also see this written as $E = h\nu$.
- https://en.wikipedia.org/wiki/Planck%E2%80%93Einstein_relation

Relativistic Quantum Mechanics

- Here we need to introduce a concept, not just from Quantum Mechanics, but from Special Relativity as it applies to Quantum Mechanics.
- While we're here, Let's talk just a bit about Planck's constant. It has an extremely small value of around 10^{-34} Joule seconds, which explains why it normally needs to be considered only in atoms and smaller. But we don't need to use this value in this presentation.
- Frequency is defined as the number of wavelengths per unit *time.* (Uh-oh, the "t" word!)
- But Time Dilation tells us that time varies with velocity. So the energy must be Lorentz transformed, meaning that E must be corrected to (using our Binomial Expansion)
  - $E * \left(1 + \frac{v^2}{2c^2}\right)$ The Relativistic Doppler Effect
- https://en.wikipedia.org/wiki/Relativistic_Doppler_effect

OK, Enough Preliminaries! The Setup

- Imagine an object T (for *thing*) floating alone in intergalactic space (i.e. no appreciable forces acting on it). It has rest mass *m*.
- Imagine that Alice is at rest relative to T (e.g. just floating next to T).
- And that Bob is in a spaceship moving at constant relative velocity *v* to the left of T (so that T appears moving to his right).
- TODO: Add picture
    - Alice is at the top, at rest
    - T is in the middle, at rest
    - Bob is on the bottom, zooming to the left at velocity v

Let There Be Light!

- Some time later, T emits two identical photons in opposite directions, each with energy E/2, parallel to Bob's path.
- TODO: 'Nother picture (same as before, but with T emitting the photons)
  - Alice at the top, as before
  - T right below her, at rest, but with two photons (probably just *'s) zooming out from it, one to the left, one to the right
  - Bob's at the bottom at velocity v
- Because the photons are equal in energy and heading in opposite directions, T doesn't move, so it doesn't change position from either Alice's or Bob's point of view.
- But because of conservation of energy, T's energy has now diminished by 2 * E / 2 = E.

Let's Calculate the Energy of T in Two Ways
Alice's Way

- Before the flash: From her POV, T isn't moving and thus has 0 kinetic energy.
- So $Energy_{Alice} = KE_{Alice} = 0$
- After the flash, T now has energy $Energy_{Alice} = KE_{Alice} – E = -E$

Let's Calculate the Energy of T in Two Ways
Bob's Way

- Before the flash: Since T is moving, it has a certain amount of kinetic energy. Call this KE_{Bob} (note that this is the kinetic energy of T, not of Bob, but it's the kinetic energy from Bob's POV);
- This is just $\frac{1}{2}\mathrm{m}v^2$, where m is the mass of object T and v is Bob's velocity relative to T.

Let's Calculate the Energy of T in Two Ways
Bob's Way

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- After the flash it's lost energy E, so its energy is
  - $\text{Energy}_{Bob} = \frac{1}{2}\mathrm{m}v^2 - E$

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  - $\text{Energy}_{Bob} = \frac{1}{2}\mathrm{m}v^2 - E$
- But wait, as we saw earlier, under relativity, the frequency of the energy must change. So according to the Relativistic Doppler Effect its energy is actually
  - $\text{Energy}_{Bob} = \frac{1}{2}\mathrm{m}v^2 - E\left(1 + \frac{v^2}{2c^2}\right)$

Who's Right – Alice or Bob?

- So Alice thinks T has energy –E
- Bob sees T having energy $\frac{1}{2}\mathrm{m}v^2 - E\left(1 + \frac{v^2}{2c^2}\right)$
- But we've never *touched* T, just observed it from a distance, neither added to nor subtracted from T's total energy.
- So the two expressions must somehow give the same result. How do we reconcile the two?

Alice and Bob Reconcile
(Isn't that sweet?)

- So we have:
- $-E = \frac{1}{2}\mathrm{m}v^2 - E\left(1 + \frac{v^2}{2c^2}\right)$

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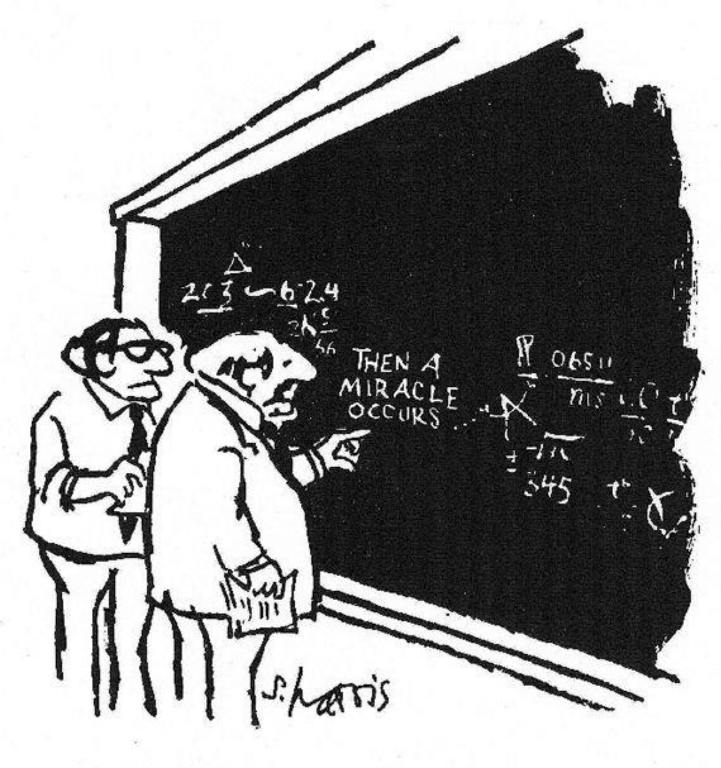
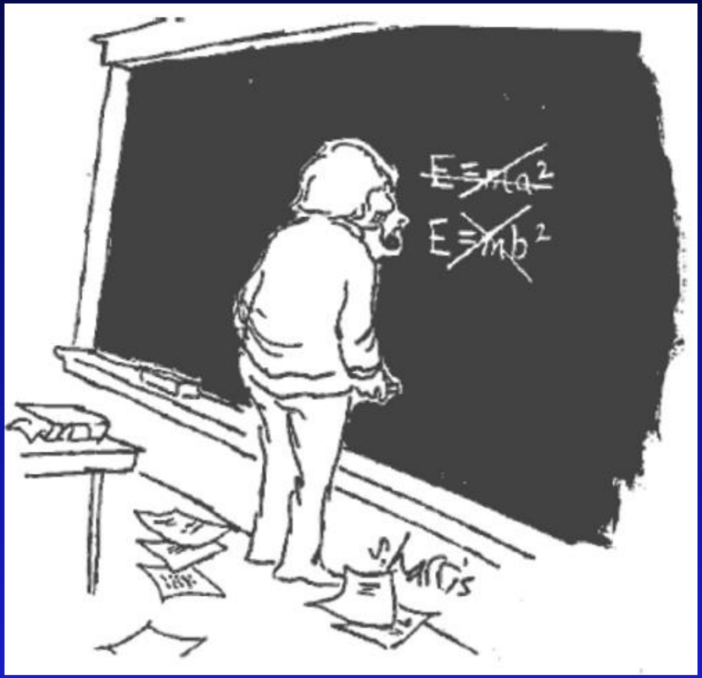
- $E = mc^2$ (TADA!)



"I think you should be more explicit here in step two."

WAIT A MINUTE!

- I thought the point of this presentation was to show that E was **NOT** equal to mc^2 !!!!!!!!!!
- But the argument we used was essentially the one Einstein used.
- So where did he go wrong? Any guesses?



Einstein's "Mistake"

- Some 30 or so slides ago, in **The Binomial Approximation of γ**, I showed that even at a speed of 1,000,000 km/sec, we got an excellent approximation to γ by using just 2 terms of the binomial expansion. Which is what Einstein used.
- But that was only an approximation. So in the **Let's Calculate the Energy of T in Two Ways – Bob's Way** slide, instead of writing $\text{Energy}_{\text{Bob}} = \frac{1}{2}\text{m}v^2 - E\left(1 + \frac{v^2}{2c^2}\right)$, I should have used the full value of γ's binomial expansion, not just the approximation.
- So the true equation is really (with $\beta = v/c$) and
  - $\gamma = 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + \frac{63}{256}\beta^{16} + \ \ldots$
- $\text{Energy}_{\text{Bob}} = \frac{1}{2}\text{m}v^2 - E(1 + \frac{v^2}{2c^2} + \frac{3}{8}\frac{v^4}{c^4} + \ldots)$
  - Here's where Einstein said "Neglecting magnitudes of 4th and higher orders…"
- And using the full expansion would lead to $E = mc^2$ + {other terms}

Let's Do It Right

- $-E = \frac{1}{2} mv^2 - E(1 + \frac{v^2}{2c^2} + \frac{3}{8}\frac{v^4}{c^4} + \ldots)$
- $-E = \frac{1}{2} mv^2 - E - E\frac{v^2}{2c^2} - E\frac{3v^4}{8c^4} - \ldots$
- $0 = \frac{1}{2} mv^2 - E\frac{v^2}{2c^2} - E\frac{3v^4}{8c^4} - \ldots$ (the –E's on both sides cancel)
- $\frac{Ev^2}{2c^2} - E\frac{3v^4}{8c^4} - \ldots = \frac{1}{2} mv^2$
- $\frac{E}{c^2} - E\frac{3v^2}{8c^4} - \ldots = \mathrm{m}$ (the $\frac{1}{2}v^2$ terms cancel)
- $\mathrm{E} = \mathrm{mc}^2 + \mathrm{m}\frac{3v^2}{8c^4} + \ldots$ (The Right Equation!)

Bottom Line

- $E = mc^2 + m\frac{3v^2}{8c^4} + \ldots$ (The Right Equation!)
- So this is why I say that E doesn't equal mc^2;
  - Note that we can't just let v = 0 since we previously divided by v^2 and you can't divide by zero.
  - But as v gets closer and closer to 0, the extra terms get arbitrarily close to zero (mathematicians call this *in the limit*) and the effect of the extra terms melts away.
- So in his *Does the Inertia of a Body Depend on Its Energy Content?* paper, Einstein knew, and knew his audience knew, that the extra terms were there and that, in the limit, E = mc^2 is true only for a body at rest (v = 0).
- So he just didn't need to spell it out in gory detail.
- But in hindsight it makes sense. If a body is in motion (v > 0), then it has extra (kinetic) energy, so of course E = mc^2 + extra.

Additional Special Relativity Features

- The next four slides describe features of Special Relativity that are interesting, but otherwise weren't needed in this presentation.
- But I assume you've heard of many/most of them, and while we're here I wanted to at least have one slide each on them.
- But since this presentation has been long enough as it is, I won't prove all of them, but will offer some hints or links to web articles that will talk about them, and try to leave some time at the end of the talk for questions.

Length Contraction

- Perhaps one of the first physics equations you ever learned was
  - d = vt (Distance = Velocity * Time)
- But as we've seen, Time is somewhat fluid. So the proper equation is actually
  - $d_{Bob} = vt / \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right) = vt / \gamma$ From Alice's point of view
- And with γ always greater than 1, the length of something decreases the faster it moves.
- So if Alice sees Bob travelling at 86.6% of c, she sees him and his whole ship shrunk to half their at-rest size.
- What this *doesn't* mean is that as we move faster, it feels like a vise is trying to crush us. Bob would feel absolutely normal.
  - Time would seem to him unchanged (since his clock is running slow).
  - His body dimensions would seem the same to him (since his ruler has been foreshortened).
- For example, at particle accelerators (e.g. CERN), if a proton is moving at almost the speed of light, it must be modeled not as a sphere but as a pancake. But the proton "thinks" it's still a sphere.

Adding Velocities

- Remember that velocity is distance / time. But we've seen that both distance and time are rather flexible. So maybe it won't surprise you that velocity isn't as simple as we've traditionally thought.
- The proof is a bit complex, so I'll just sketch things. Search for <relativity velocity addition>.
- But the idea behind this formula uses the Lorentz contraction twice, once for each velocity.
- If you have two objects travelling at speeds *u* and *v* (e.g. our 100 km/hr train and our 80 km/hr baseball), their combined speed will <u>not</u> be 180 km/hr. The formula is
- $u + v = \frac{u+v}{1+\frac{uv}{c^2}}$ (or $u - v = \frac{u-v}{1-\frac{uv}{c^2}}$ if the objects are heading towards each other)
- For our baseball, this comes out to 179.99999999998397 km/hr.
- For our flashlight, u = 100 km/sec and v = c = 300,000 km/sec (or so).
- v = c : Plug v = c into the formula above. Any value of u will give the answer c. This confirms Maxwell's assertion that the speed of light is the same in all inertial frames.
- http://math.ucr.edu/home/baez/physics/Relativity/SR/velocity.html (Side note: John Baez is Joan's cousin!)

Mass Increases With Velocity

- From high-school physics, momentum = mass * velocity
  - p = mv
- Momentum can be large if the mass is large but the velocity is small (e.g. a freight train slowly wending its way). Or it can be large if the mass is small but the velocity is large (e.g. a rifle bullet). Or some combination of the two.
- Basically, momentum tells us how resistant to acceleration something is.
- As we now know, whenever we see v (velocity) in a formula, relativity tells us that it really should be γv. So it's really p = γmv. Let's write this as p = (γm)v
- Since $\gamma > 1$, the faster the velocity, the effective mass ("relativistic mass") increases. So the momentum increases and it makes it harder to accelerate the object. And as v approaches c, γ tends towards infinity and it would require an infinite amount of energy to reach light speed.
- https://www.youtube.com/watch?v=LTJauaefTZM
- https://www.youtube.com/watch?v=A2JCoIGyGxc

Arguably…

- Here's an argument that might make time dilation a bit more intuitively plausible.
- Imagine being in a car on a large Utah salt flat. You're at point A, due south of point B, say, 1 kilometer away. Your car always drives at the same speed, say, 100 km/hour.
- Point C is, say, 10 km to the East of B.
- You can drive your car directly from A to B. This takes the least amount of time, but you don't travel any distance East.
- You can drive your car (at 100 km/hr) from A to C. You'll spend more time, but you'll cover more distance Eastward.
- Spacetime is like this. You can stick in one place and you'll get to the future as fast as possible. But if you start moving, you'll be trading distance for time (it'll take longer).
- And realize that points A, B and C define a right-angled triangle. Sound familiar?

Thank You

- This presentation can be found at
    - http://lrs5.net/FTPData/NoEDoesNotEqualmc2.pdf

TODO: Possible New Slides

- Feedback while developing this talk showed that some people were uncomfortable with all the math. Most of it was deliberately kept at the high-school level, but for some the ability to follow it had decayed through lack of use.
- The problem is that there are just way too many popular science books that *tell* you about Relativity, without providing real insight into what's going on under the covers.
- But to support the "… and I'll Tell You Why" aspect of the talk's title, you absolutely need to present the algebra.
- So the guts of the talk will be presented in two sections. The first will present the concepts in as much detail as possible, doing the minimum amount of math but still trying to get the audience comfortable with what's going on.
- After that we'll go back and re-present the same ideas, this time with equations.