Higher Dimensions Of Space

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My Credentials

Questions
During the Presentation

- I will certainly entertain questions being asked during the talk.
- But we have a lot of ground to cover and if the answers take too long, or get too far afield, we'll make a note of the question and field it later. So please understand if I try to stay on topic.
- But please do ask me to stop and clarify things, if necessary. I will be going pretty fast.

How Many Dimensions Are There in the Universe?

- Audience poll…
- How many spatial (i.e. not including time) dimensions do you think there are?
- I just knew someone was going to say 42!

Relativity

- In 1905, Einstein published his Special Theory of Relativity.
  - “Special” because it limited itself to the special case of objects with constant velocity.
  - Newton’s concept of fixed space and fixed time were overthrown.
- In 1915 he finished his General Theory of Relativity.
  - “General” because he now was able to analyze the general case in which objects could have non-constant velocity (i.e. they could accelerate).
  - This had the surprising implication that spacetime wasn’t rigid, but could (and must, in the presence of matter) curve.

To Infinity and Back Again (taking the long way home)

- The surface of a sphere is what's known as "finite but unbounded", meaning you can walk forever on its finite surface but never come to a boundary.
- An old science fiction idea is that the universe is finite but unbounded and that if you set out in one direction and kept going for a very long time, you'd come back to where you started.
  - This has since been disproved by an analysis of the Cosmic Background Radiation.
- This raises the possibility -- Are the the dimensions of space finite but unbounded?

Theodor Kaluza

- German/Polish mathematician and physicist.
- In 1917, sent Einstein a copy of his paper about reformulating GR in five dimensions!
- Not surprisingly, to "stretch" GR and have it fit "snugly" into 5-D, some extra conditions (i.e. equations) had to be satisfied.
- Amazingly, these turned out to be exactly Maxwell's equations that govern electromagnetism!

Think About It

- You start with a 4-dimensional universe. You know of only one force in that universe (called *gravity)* and you have a theory that describes it (General Relativity).
- You reformulate it in 5 dimensions and a brand new force appears!
- And it's electromagnetism!!!
- And who would have thought that by considering the existence of another dimension you would find that electromagnetism is a necessary consequence of gravity?

Einstein's Reaction

- "The idea of achieving [a unified theory of gravity and electromagnetism] by means of a five-dimensional cylinder world would never [have] dawned on me. … At first glance I like your idea enormously."
- But he still held out until 1921 to endorse Kaluza's paper.

Now Where Did I Misplace that Extra Dimension?

- In 1926, Oskar Klein simplified Kaluza's theory and also was able to remove some limitations of the original theory. Thus it's now called the Kaluza-Klein theory.
- But he also did some calculations that hinted that the reason you couldn't see the 5th dimension was that it was only about 10^{-31} meters in size. It was a "compacted" dimension.
  - The LHC will probe down to about 10^{-19} meters.
- An open question – if there are other dimensions, why are some large and some small?
- Next up: Just what *is* electric charge?

Is Electric Charge a 5^{th} Dimensional Phenomenon?

- In 5-D, the gravitational force was modified if particles were moving in the new dimension.
- This new force was identical to the electric force.
- And the electric charge was nothing but the component of momentum in the 5th direction.
- If two particles cycled in the same direction around the compact space, they repelled each other.
- If they moved in the opposite direction, they attracted.
- If either of them did not cycle in the compact direction, then only ordinary gravity affected them.

The Bad News

- Kaluza's theory, while provocative, was shown not to fully describe the real world.
- Which isn't surprising, since there were forces and particles left to be discovered.
  - e.g. the neutron wasn't discovered until 1932.
- Perhaps it failed because Kaluza assumed the extra dimension was an exact circle and maybe that's not general enough. And maybe 5 dimensions wasn't enough.

- Remember, the size of the 5th dimension might be 10^{-31} meters.
- Atoms are about a tenth of a nanometer (10^{-10} meters) in size.
- Thus they're about 10^{21} times larger than the postulated size of the 5th dimension.
- If the 5th dimension is finite but unbounded, then if I could move the tip of my finger as little as the size of one atom, I'd have jogged around the 5th dimension 10^{21} (1,000,000,000,000,000,000,000) times!
  - And they say I don't get enough exercise!

String Theory
Get the Point?

- The Standard Model of Physics assumes that electrons are point particles. But this leads to divide-by-zero problems.
- Gravity (Newton) – $F = Gm_1m_2/r^2$
- Coulomb's Law – $F = k_eq_1q_2/r^2$
- As the particles get ultimately close together, you wind up with $r = 0$, which means the equations want you to divide by zero. Which is mathematically undefined and thus meaningless.

Effective Theories

- We know that Newton's law of gravity is wrong. For example, if the GPS satellites used it, it would give the wrong locations. Instead they use General Relativity.
- Yet NASA can (and does) use Newton's equations to navigate to he moon and to the rest of the solar system.
- But how can we do this if the theory is wrong?
- We don't (yet!) know the ultimate laws of nature. So we have to make do with laws that are approximate, that are applicable over a range.
- These are called *effective* theories. Newton is fine as long as speeds are small compared to the speed of light and don't involve strong gravitational fields.
- Similarly, General Relativity we know is wrong if we assume that it makes sense to talk about objects arbitarily close to each other. That again leads to divide-by-zero problems. So GR is also just an effective theory.

The Nonsense of Singularities

- I'm personally annoyed that so many otherwise respectable physicists have written books and/or appeared on TV science documentaries, and blithely refer to "singularities", throwing around phrases such as "infinite density at the center of black holes".
- Maybe they're just trying to (over-) simplify things.
- But the truth is that General Relativity is only an effective theory. It breaks down when the distances involved become too small.
- ***There are no such things as singularities!***

String Theory
Get the String!

- Starting in 1970, string theory was invented. It posits that all particles are tiny (but not zero sized) strings of energy.
  - They can be open (a string with two ends) or closed (a loop).
- Just as a violin string can produce many notes, depending on how it vibrates…
- All particles (if string theory is right) are strings vibrating in different ways.
- If it vibrates this way, it's an electron. If it pulsates that way, it's a quark. And so on.
- So this addresses the issue of electrons maybe not being point particles.

Strings

The Original String Theory

- The first version of string theory was a quantum theory that modeled only bosons (particles of energy, e.g. the photon).
- To model fermions (matter particles), it needed a new concept. See later.
- It was (had to be) formulated in 26 dimensions.
- Perhaps most importantly, the equations required a new particle. This was later shown to be the graviton.
  - String theory predicts gravity!
- String theory was a quantum theory of gravity!

Just a Bit of Symbology

- In the next slide we'll use the Σ (the Greek letter sigma) notation to indicate summation for (in this case) for all values of i, starting at i=1, then i=2 and continuing for all values up to infinity (∞).

$$\sum_{i=1}^{\infty} formula_i$$

- This is the same as
- $formula_1 + formula_2 + formula_3 + \ldots$
- So if the formula were as simple as the square root function, this would be
- $\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots$

How Many Dimensions?!?

- *Very* roughly speaking, the following calculation appeared in the original string theory.

$$\sum_{i=1}^{\infty} formula_i \times \left(1 - \frac{1}{24}(D-2)\right)$$

- where D is the number of dimensions we're talking about.
- This calculation evaluated to ±∞, *unless* D = 26!
- String theory ***predicts*** the number of dimensions!

Enter Supersymmetry

- Remember, the original string theory modeled only bosons (force), not fermions (matter).
- The concept of *supersymmetry* (SUSY) proposed a hitherto unsuspected relationship between the two.
- Every boson has a fermion partner, and vice-versa.
- This doubles the number of particle types in the universe!
- If supersymmetric particles exist, they will probably (hopefully!) be detected by the LHC.
- Sparticles are candidates for dark matter.

Particles and Sparticles

Normal Particle	Superparticle
Electron	Selectron
Quark	Squark
Top Quark	Stop Squark (I kid thee not!)
Neutrino	Sneutrino
Photon	Photino
Graviton	Gravitino
W particle (Weak force carrier)	Wino (sorry, but this is pronounced "weeno")
etc	

Failure then Success

- Superstring theory was at first snubbed by most physicists.
- It didn't really describe our universe that well.
- In particular, some of its basic formulas predicted infinite values.
- But in 1984, in what's termed "The First Superstring Revolution", Michael Green and John Schwarz resolved some of the worst mathematical problems.
- Suddenly, there was a lot of interest in the theory.

There and Back Again

- So in theory we can turn a particle/sparticle into a sparticle/particle and back again.
  - For example, an electron can emit a photino and turn into a selectron, and vice-versa.
  - So we would have: electron → selectron → electron.
- And we're back where we started.
- Not quite!
- The original electron will now be *shifted* in space.
- Is there a tie-in to gravity here? Maybe.

Superstring Theory

- Merging the concept of supersymmetry into the original string theory produced what we now call superstring theory.
- So, technically, string theory ≠ superstring theory.
- But since the original string theory is essentially passé, we now normally just refer to string theory but mean superstring theory.

We're Doing Better(?) With the Number of Dimensions

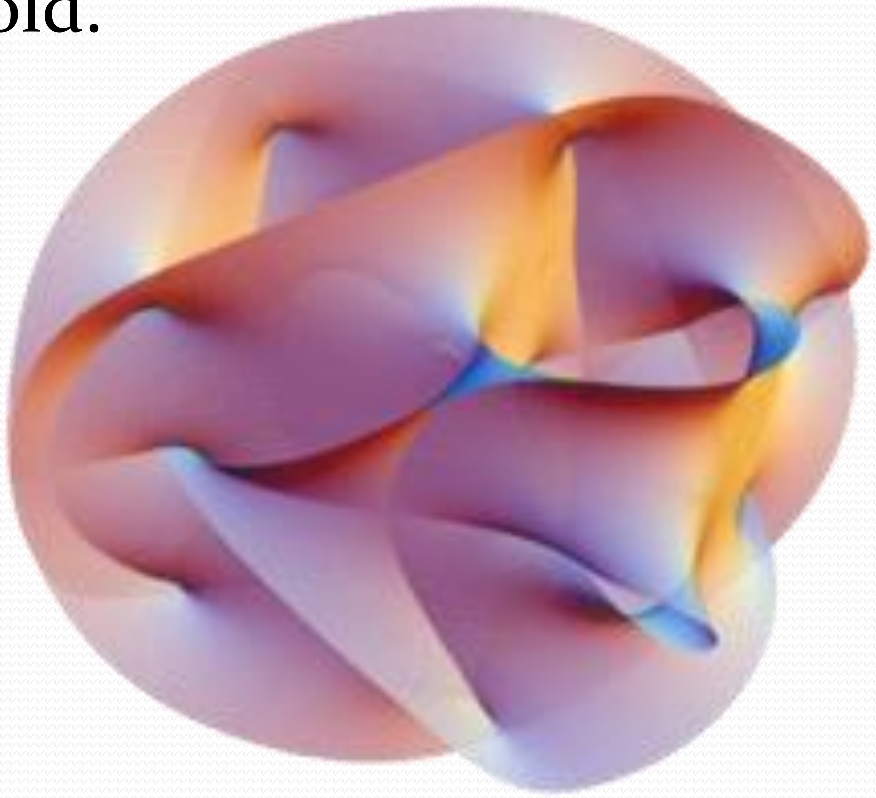
- Whereas the original string theory required 26 dimensions, superstring theory requires only 10.
  - i.e. the updated equations in superstring theory are consistent only if D = 10.
- ~~*This is an improvement???*~~
- This is an improvement!!!
  - Well, somewhat better, at least.

What Do These Other Dimensions Look Like?

- Kaluza's original 5th dimension was a simple circle.
- String theory requires that the extra 6 dimensions be curled up ("compactified") in highly specific ways.
- The good news is that mathematicians had already investigated these.
- They're called Calabi-Yau spaces.
  - Eugenio Calabi and Shing-Tung Yau.

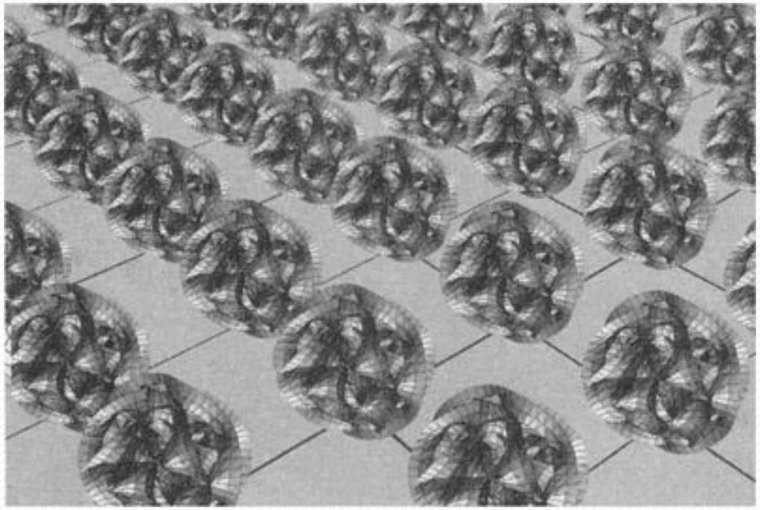
Picture of a Calabi-Yau Space

2-D flattening of 3-D projection of 6-D Calabi-Yau manifold.



Where Are They?

- Answer – At every single point in our normal concept of space.



- Shown spaced out, else the picture would be entirely black!

Calabi-Yau Spaces – What Good Are They?

- They're ultra-small. They're convoluted. Can't we just ignore them?
- The Standard Model is incomplete. There are some two dozen constants of nature that aren't predicted by theory and must be put in by hand (i.e. from experimental data). Among others, these include…
  - Masses of all the elementary particles (electron, quarks, etc).
  - Strengths of forces of nature (electricity, gravitation, etc).
- The precise shape of the Calabi-Yau manifold may determine these.
- One problem – there may be 10^{500} of them! Maybe even 10^{1000}!!!
  - Which may actually turn out to be a good thing!

Calabi-Yau Implication #1
The Universe and Goldilocks

- If the strong nuclear force (that keeps nuclei together) were a few % stronger or weaker, fusion reactions would change and either stars wouldn't form at all, or they would burn out quickly before life could form on their planets.
- If the charge on the electron were off by a few %, either atoms could not retain electrons, or else electrons would not bond with other atoms. Either way, no molecules.
- And there are other examples. But all say that if the constants of nature were only a few % different, life as we know it would not exist.

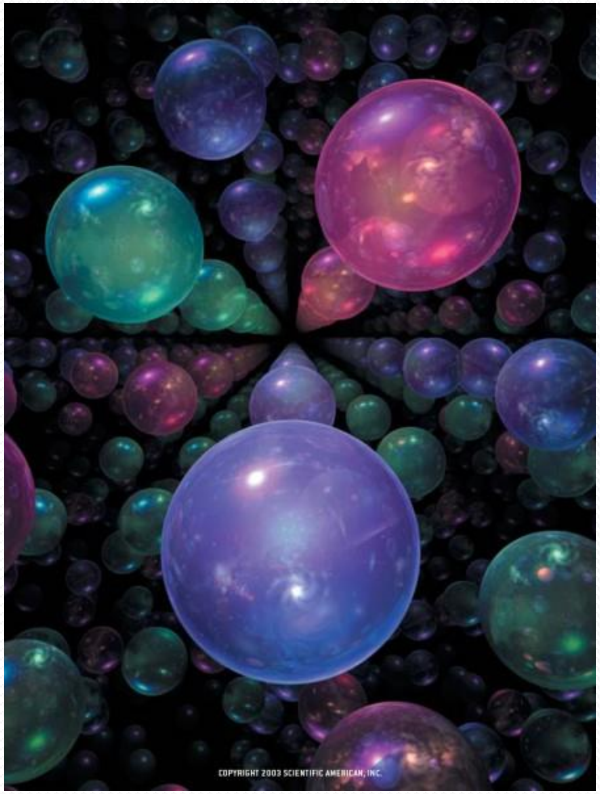
The Anthropic Principle

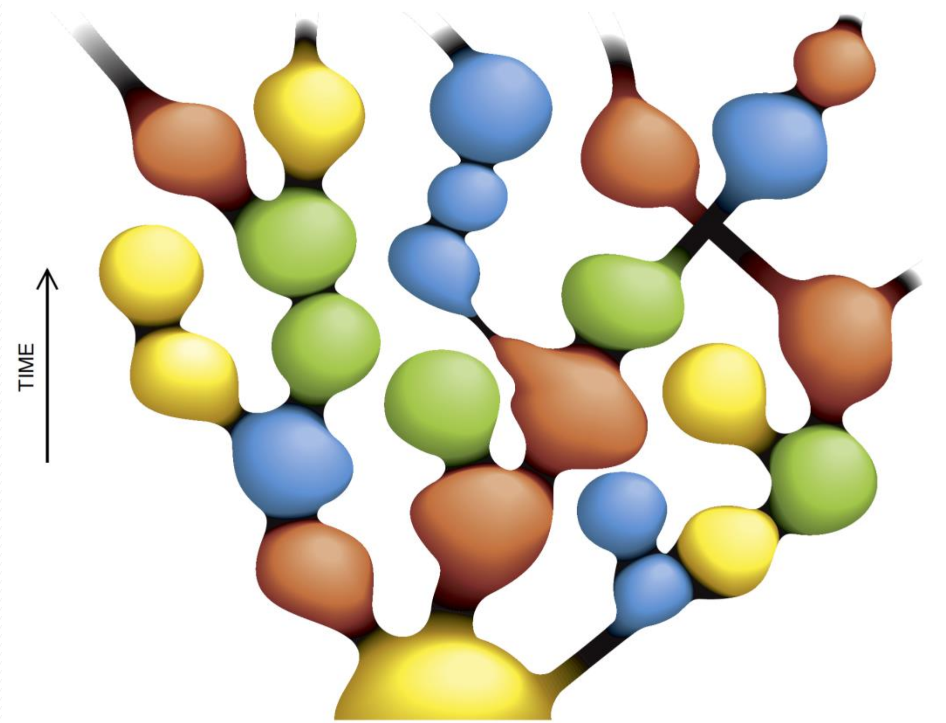
- There are several versions of this, but for our purposes, the Anthropic Principle says that…
- The laws of nature, and its fundamental constants, must be such as to allow observers such as us to be present.
- And as we've just seen, there are *very* tight constraints on this.
- So why is the universe so fine-tuned?

Calabi-Yau to the Rescue!

- There is reason to believe that at the Big Bang, multiple universes were created – a Multiverse!
- In any given universe, all points would have the same Calabi-Yau shape.
- But different universes could have different Calabi-Yau shapes, and thus different laws of physics with different constants of nature.
- So the vast majority of these universes are probably sterile.
- But by the Anthropic principle, it's no surprise that we're in one of the relatively few universes that are fine-tuned for life.

The Multiverse

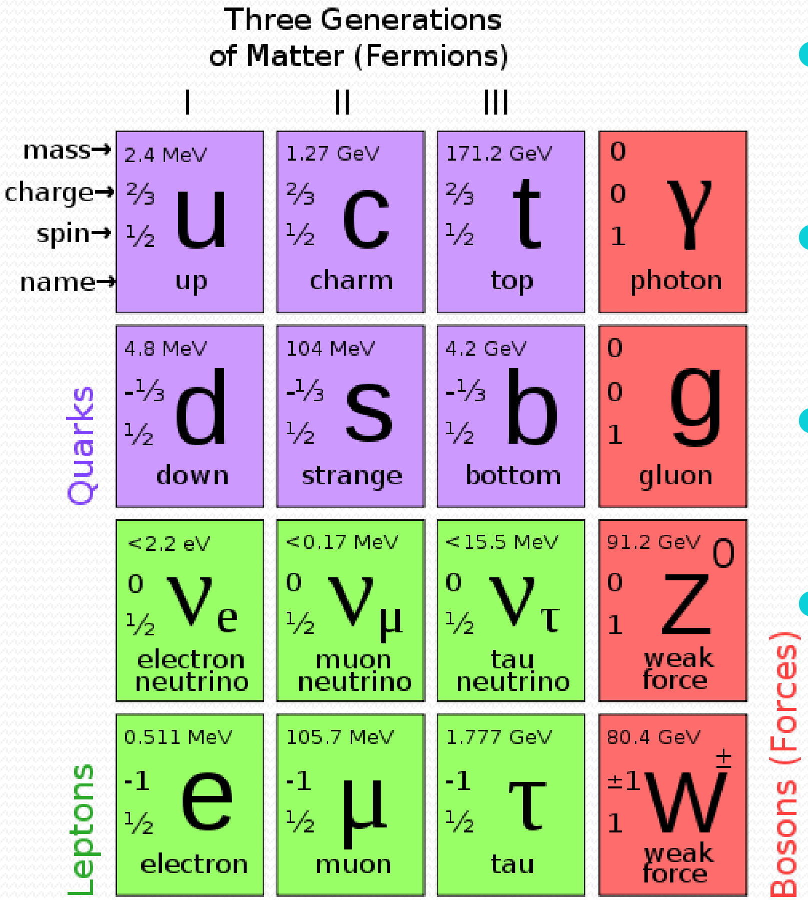




Calabi-Yau Implication #2
Hol(e)y Calabi-Yau!

- Matter seems to come in 3 families (also known as generations).
  - Electrons, muons and tau particles
    - Each with their associated neutrinos.
  - Up/Down, Strange/Charm, Top/Bottom quarks.
- A Calabi-Yau space with 3 (multidimensional) holes could explain this.
- Some CY shapes have been found that give just the right number of force particles as well as just the right electric and nuclear force properties to match the particles.

String Theory : Generations



- Each of the first 3 columns is a family.
- The 4th column shows the particles of force.
- Other particles are made up of quarks (e.g. the proton = uud).
- Missing – antiparticles, the Higgs boson(s), the graviton, sparticles and maybe others.

Suggested Reading

- Subtle is the Lord, by Abraham Pais.
  - A biography of Einstein. Most biographies concentrate on the facts that he played the violin, that he met Charlie Chaplin, that he was offered the presidency of Israel, etc. But this is the biography I think he would have liked best. It concentrates on his scientific life. And when you get to the tensor equations embedded in the text, just *bleep* over them.
  - He has other books on Einstein, Bohr and Teller. I particularly liked *Inward Bound*, a history of the early days of particle physics (and physicists).

Suggested Reading

- Warped Passages, by Lisa Randall.
  - She was the first tenured woman in the Princeton University physics department and the first tenured female theoretical physicist at MIT and Harvard University.
  - Her specialty is creating mathematical models of new concepts in physics, often involving higher dimensions.
  - Watch the video of her being interviewed on PBS by Charlie Rose at http://video.google.com/videoplay?docid=-45154219728824809&q=tvshow%3ACharlie_Rose&hl=en#

Suggested Reading

- The Elegant Universe, by Brian Greene.
  - A very readable introduction to modern string theory. It gets a bit technical when he talks about his (to him) big discovery, but you forgive a lot when the rest of the book is this good.
- The Fabric of the Cosmos, by Brian Greene.
  - Like his previous book, but not limited to just string theory.
- http://www.youtube.com/watch?v=YtdE662eY_M

Suggested Reading

- The Road to Reality, by Sir Roger Penrose.
  - At over 1,000 pages, this is an absolutely amazingly comprehensive book on practically all aspects of particle physics, cosmology, quantum mechanics and more.
  - There's lots of readable prose, but there's also a lot of mathematics thrown in for those who can handle it.
  - But even if you can't hack the math, there are enough fascinating nuggets in the prose that it's a treasure trove of information for the physics enthusiast.

Suggested Reading

- Flatland, by Edwin A. Abbot
  - A classic, written in 1884. It tries to help us (more or less) 3-dimensional beings understand the 4^{th} dimension by imagining how a 2-D being might envision a 3-D world.
- Sphereland, by Dionys Burger
  - A pastiche sequel to Flatland, written in 1965.
  - Set in Flatland, it introduces us to the concept of curved space (e.g. their scientists find out that the sum of the angles in a triangle is > 180º).
- The two novels are available as a twofer.

Suggested Reading

- The Cosmic Landscape (String Theory and the Illusion of Intelligent Design), by Leonard Susskind.
  - Susskind is one of the originators of string theory. In this book he talks widely about extra dimensions, Calabi-Yau spaces, branes, the Anthropic Principle, etc, etc, etc.
    - YouTube has many of Susskind's lectures at Stanford. Just search it for *Susskind*. Note that most of these are university-level lectures, but you can probably get some ideas from them.
    - His lectures are also available on iTunes.
    - See http://en.wikipedia.org/wiki/Leonard_Susskind for a list of them.

Suggested Reading

- The Shape of Inner Space: String Theory and the Geometry of the Universe's Hidden Dimensions, by Shing-Tung Yau.
  - As in Calabi-Yau
  - Not the best written popular science book I've ever read. Probably best obtained from the library.

Suggested Reading

- The Trouble With Physics, by Lee Smolin
  - The other point of view. There are other candidates for a quantum theory of gravity (such as Causal Dynamic Triangulations and Smolin's own Loop Quantum Gravity), which need only 4 dimensions.
  - This book explores the weaknesses of string theory and criticizes the academic world for being too "faddish" on the subject, and discouraging research into alternatives.
  - As with all (well, most) of Smolin's books, it's well written and interesting.

Thank You