Higher Dimensions Of Space Part II

The universe is not only stranger than we imagine; it is stranger than we can imagine.

J.B.S. Haldane, 1927

Larry Smith

My Credentials

My Credentials

- I have none. I'm neither a physicist nor any kind of scientist.
- I'm a computer nerd with a lifetime interest in math and fundamental physics.
- So while I think I've done my homework correctly and what follows is a correct description of string theory, you should take everything I say with a gain of salt.

The Mistake I'm About to Make

- They say that the worst thing a presenter can do is to simply read what's on the slide.
- Which is annoying, because then, who needs the presenter? People can just read the slides! They want additional commentary.
- But I've put a lot of work into condensing concepts down to hopefully comprehensible bullet points that people can study later and perhaps understand better.
- So sometimes I can't do better than just read some of the points. Hope you don't get too annoyed when I do!

Disclaimer

- While String Theory (ST) is provocative, it currently has no experimental evidence to support the theory.
- In particular, ST's requires more than our usual 4 dimensions (length, width, height, time). This is awfully non-intuitive.
- But from Special and General Relativity, we know that our everyday concepts of space and time are much more flexible than we would naively think. So maybe extra dimensions aren't totally out of the question

Fermilab Videos Highly Recommended

- https://www.youtube.com/results?search_query=f ermilab
 - Tons of great videos here
- Big Mysteries: Extra Dimensions https://www.youtube.com/watch?v=5UDUNqwWuNs
- What is Supersymmtery -https://www.youtube.com/watch?v=oCeLRrBAI6o
- What does the Muon g-2 experiment tell us? https://www.youtube.com/watch?v=eCCGr4BqElE&list=PLCfRa7MXBEsoJuAM8s6D8oKDPyBepBosS&index=12

String Theory on YouTube

- If you had to look at just one page for string theory (pro and con), check out all the links at https://www.youtube.com/results?search_query=why+string+theory+is+right
- Especially
 - Why String Theory is Wrong -https://www.youtube.com/watch?v=IhpGdumLRqs
 - Why String Theory is Right https://www.youtube.com/watch?v=iTTa9YcTe1k&t=3s

- The Elegant Universe, by Brian Greene.
 - A very readable introduction to modern string theory. It gets a bit technical when he talks about his (to him) big discovery, but you forgive a lot when the rest of the book is this good.
- The Fabric of the Cosmos, by Brian Greene.
 - Like his previous book, but not limited to just string theory.
- The Hidden Reality, by Brian Greene
 - Chapter 9 Black Holes and Holograms
- http://www.youtube.com/watch?v=YtdE662eY_M

- Warped Passages, by Lisa Randall.
 - She was the first tenured woman in the Princeton University physics department and the first tenured female theoretical physicist at MIT and Harvard.
 - Her specialty is creating mathematical models of new concepts in physics, often involving higher dimensions.
 - Watch the video of her being interviewed on PBS by Charlie Rose at

https://www.youtube.com/watch?v=tgcDbJI4f-M

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- Subtle is the Lord, by Abraham Pais.
 - A biography of Einstein. Most biographies concentrate on the facts that he played the violin, that he met Charlie Chaplin, that he was offered the presidency of Israel, etc. But this is the biography I think he would have liked best. It concentrates on his scientific life. And when you get to the tensor equations embedded in the text, just *bleep* over them.
 - Pais has other books on Einstein, Bohr and Teller. I particularly liked *Inward Bound*, a history of the early days of particle physics (and physicists).

- The Road to Reality, by Sir Roger Penrose.
 - At over 1,000 pages, this is an absolutely amazingly comprehensive book on practically all aspects of particle physics, cosmology, quantum mechanics and more.
 - There's lots of readable prose, but there's also a lot of mathematics thrown in for those who can handle it.
 - But even if you can't hack the math, there are enough fascinating nuggets in the prose that it's a treasure trove of information for the physics enthusiast.

- The Physical World, by Nicholas Manton and Nicholas Mee
 - Reminiscent of *The Feynman Lectures On Physics*, this university-level book runs over 500 pages long and is crammed with excellent detailed descriptions of the physical world, including Newton's Laws, Thermodynamics, Astronomy, Maxwell's equations, Special and General Relativity, Quantum Mechanics, Particle Physics, and more.

- Flatland, by Edwin A. Abbot
 - A classic, written in 1884. It tries to help us (more or less) 3-dimensional beings understand the 4th dimension by imagining how a 2-D being might envision a 3-D world.
- Sphereland, by Dionys Burger
 - A pastiche sequel to Flatland, written in 1965.
 - Set in Flatland, it introduces us to the concept of curved space (e.g. their scientists find out that the sum of the angles in a triangle is > 180°).
- The two novels are available as a twofer.
 - https://www.amazon.ca/Flatland-Sphereland-Edwin-Abbott/dp/oo62732765/ref=sr 1 1?crid=GG1B8R14UWWK&keyword s=flatland+sphereland&qid=1645913009&sprefix=flatland+spherelan d%2Caps%2C41&sr=8-1

- Hiding in the Mirror, by Lawrence M. Krauss
 - Probably better know for his book, *The Physics of Star Trek*, Krauss is an award-winning particle physicist and cosmologist at Arizona State University.
 - Subtitled The Mysterious Allure of Extra Dimensions, from Plato to String Theory and Beyond.

- The Cosmic Landscape (String Theory and the Illusion of Intelligent Design), by Leonard Susskind.
 - Susskind is one of the originators of string theory. In this book he talks widely about extra dimensions, Calabi-Yau spaces, branes, the Anthropic Principle, etc, etc, etc.
 - YouTube has many of Susskind's lectures at Stanford. Just search it for *Susskind*. Note that most of these are university-level lectures, but you can probably get some ideas from them.
 - His lectures are also available on iTunes.
 - http://en.wikipedia.org/wiki/Leonard_Susskind for a list of them.

- The Inflationary Universe, by Alan Guth.
 - This theory (since partially confirmed by an analysis of the Cosmic Background Radiation) posits that starting only 10⁻³⁶ seconds after the Big Bang, and lasting until 10⁻³² seconds, a chunk of space expanded by a factor of 10⁷⁸! This became our universe.
 - Other chunks could have expanded into a different universe, and the laws of physics there could be radically different from those in our universe. This is one aspect of the Multiverse theory.

- The Shape of Inner Space: String Theory and the Geometry of the Universe's Hidden Dimensions, by Shing-Tung Yau.
 - As in Calabi-Yau
 - Not the best written popular science book I've ever read. Probably best obtained from the library.

Suggested Reading (Off Topic)

- This book is off-topic, but is too good not to plug!
- Black Holes and Time Warps (Einstein's Outrageous Legacy), by Kip Thorne.
 - From *Publisher's Weekly* -- Thorne, the Feynman Professor of Theoretical Physics at CalTech, here offers an accessible, deftly illustrated history of curved spacetime. Covering developments from Einstein to Hawking, he takes his readers to the very edge of theoretical physics: straight through wormholes--and maybe back again-past hyperspace, "hairless" wormholes and quantum foam to the leading questions that drive quantum physics. He even addresses the tabloid taunt that has tantalized him since 1988: Do quantum laws allow time travel? (In his foreword, Hawking suggests, "Maybe someone will come back from the future and tell us the answers.") Thorne is rigorous, modest and, true to the spirit of science, determined that readers move beyond the appeal of exotic answers and grasp the significance of quantum questions. This volume, a model of style, format and illustration, will speak eloquently to the readership, ranging widely in scientific literacy and interest, that such theoretical physics writers as Hawking and Feynman have established.

- The Trouble With Physics, by Lee Smolin
 - The other point of view. There are other candidates for a quantum theory of gravity (including Causal Dynamic Triangulations and Smolin's own Loop Quantum Gravity), which need only 4 dimensions.
 - This book explores the weaknesses of string theory and criticizes the academic world for being too "faddish" on the subject, and discouraging research into alternatives.
 - As with most of Smolin's books, it's well written and interesting.

Perimeter Institute Public Lectures

- From 2008 -- What Banged? by Neil Turok https://pirsa.org/08030033
- From 2014 Surprises in Supersymmetry by Dr. Jim Gates, Jr. -- https://www.youtube.com/watch?v=uUrLDh_dMHw
- From 2015 -- String Theory: A Symphony of Nature by Dr. Amanda Peet (UofT) --https://www.youtube.com/watch?v=9XuaA9HmGZ4
- From 2015 -- String Theory Legos for Black Holes by Dr. Amanda Peet (UofT) -- (1651) A.W. Peet Public Lecture:
 String Theory Legos for Black Holes YouTube

The Wolfram Physics Project

- A totally different approach to this whole topic is presented at https://www.wolframphysics.org/
- The concept is simple. Assume that the most fundamental things (I deliberately don't call them "particles") have rules about how they interact. Try out different rules and see what you get and see how close you can get to describing our universe.
- See https://www.wolframphysics.org/technical-introduction/ for more details.
- As a sort of introduction to it, check out https://en.wikipedia.org/wiki/Conway%278 Game of Life
 - The "glider gun" animation, constantly producing new "gliders", makes me think of the Big Bang.

Summary of Part I Kaluza-Klein Theory

- In 1921, Theodor Kaluza (with later revisions by Oskar Klein) reformulated General Relativity in *five* dimensions and wound up *predicting* electromagnetism.
- In this theory, electric charge was momentum along the new axis.
- The 5th dimension was unseen because it was calculated to be only some 10⁻³¹ (later revised to 10⁻³⁵) meters in size.
- This theory, while provocative, didn't match up with experiment.

Summary of Part I Bosonic String Theory

- String theory (1970) treats all particles as vibrating strings of energy.
- Originally it was a theory of bosons (force particles) but did not handle fermions (matter particles).
- It worked only in 26 dimensions and required a graviton particle.
- It was the among the first quantum theories of gravity!
- This was a major step forward. Physics has yet to reconcile General Relativity and Quantum Mechanics.

Summary of Part I Superstring Theory

- String theory was later modified to handle fermions by incorporating the concept of supersymmetry.
- The new theory was called superstring theory.
 - These days, when we refer to string theory, we mean superstring theory.
- This new theory required only 10 dimensions, not 26.

Summary of Part I Sparticles

- Supersymmetry postulated that every normal particle (boson or fermion) had an associated partner particle (a fermion or a boson) called, generically, a sparticle.
 - We've seen this before. Every particle has an anti-matter version.
- So there is (if supersymmetry is correct) a new (s)particle, the partner of the electron (a fermion), called the selectron (a boson).
- An electron can be transformed into a selectron and back again.
- And similarly for all other particles.
- These may be part of dark matter.
- But note: Sparticles have never (yet) been experimentally observed.

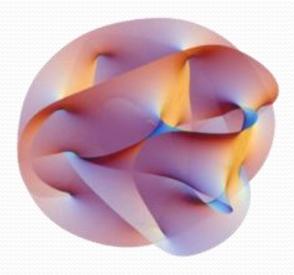
Some Particles and Sparticles

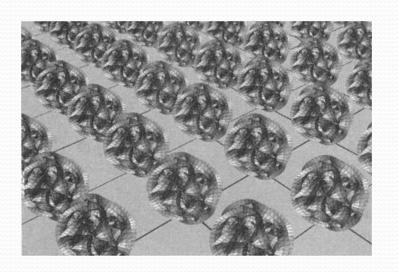
Normal Particle	Superparticle
Electron	Selectron
Quark	Squark
Top Quark	Stop Squark (I kid thee not!)
Neutrino	Sneutrino
Photon	Photino
Graviton	Gravitino
W particle (Weak force carrier)	Wino
Higgs	Higgsino

Summary of Part I Calabi-Yau

The other 6 dimensions are curled up into a Calabi-Yau space, each about 10⁻³⁵ meters in size.

These extra dimensions are at every point in our normal concept of space.





Summary of Part I Calabi-Yau Explanatory Power

- Our current best description of the universe (the Standard Model) hasn't been able to answer many reasonable questions, such as why the mass of the electron is what it is, why gravity is so much weaker than electromagnetism, why matter comes in 3 generations, and so forth.
 - Generations:
 - Leptons Electron, muon, tauon + their respective neutrinos.
 - Quarks up/down, strange/charm, top/bottom.
- But the exact shape of the Calabi-Yau space could answer these questions and more.
- For example, a 3-dimensional "hole" might explain why matter comes in 3 generations.
- However, we don't know the exact shape.

Summary of Part I The Goldilocks Universe

- Calabi-Yau spaces can be wrapped up in perhaps 10⁵⁰⁰ ways (maybe 10¹⁰⁰⁰!).
 - Some say that superstring theory loses all predictive power with an essentially infinite number of possible universes.
- But our universe is finely tuned. If the forces of nature were just a few percent different, life could not exist.
- The Multiverse theory says that there are many universes, each with a different Calabi-Yau shape, and most with the wrong parameters for life.
 - But with 10⁵⁰⁰+ possibilities, we would be in one of the universes that was "just right".

OK, Now on to the New Stuff!

An Embarrassment Of Riches

- String theory had a problem.
 - There's *one* theory of gravity.
 - There's one theory of electromagnetism.
 - There's *one* theory of quantum mechanics.
- There were *five* superstring theories!
 - Each one using string theory to describe the universe from different points of view, and each self-consistent.
- That's not good. We'd prefer a *unique* theory of the universe.

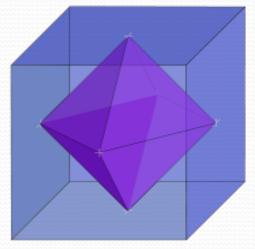
The Five String Theories

- Type I
- Type IIa
- Type IIb
- Heterotic ("hybrid vigor") SO(32), based on the mathematical symmetry group SO(32)
- Heterotic E, based on symmetry group E8 x E8

As an example of how these are different theories, Type I describes both open and closed strings, while the others describe only closed strings.

Duality

- Sometimes in math and physics, there are two (seemingly) totally different ways to describe the same thing.
- For example, take a cube and connect the midpoints of each side.
 You get an octahedron.
 - And vice-versa.
 - http://georgehart.com/virtual-polyhedra/duality.html#:~:text=So% 20the%20dual%20of%20the%20cub e%2C%20%7B4%2C%203%7D%2C,0 ctahedron%20and%20see%20that% 20it%20is%20a%20cube.
- In other words, they are two descriptions of the same mathematical object.



- At times you can prove theorems about an octahedron more easily if you think about it as a kind of cube.
 - And vice-versa.
- Electricity and magnetism are also dual.
- So are particles and waves.
- Last minute addition:
 https://phys.org/news/2022-06-duality-physics-mystery.html
- We'll see several other dualities in later slides.

The Second Superstring Revolution

- In 1995, Ed Witten (drawing on the work of others) stunned the physics world by showing that there was an 11th dimension that should be incorporated into string theory and was dual with 10-D string theory.
 - Previous analyses made a mathematical simplification that implicitly assumed that an extra dimension was zero in size.
- He called this M-Theory, where "M" stands for Mystery, Mother, Membrane, etc.



One Theory To Rule Them All

- Each of the previous 5 string theories was shown to be a version of the more general M-Theory.
- They were related to each other by several dualities, known as S-Duality, T-Duality and the 10-D/11-D duality just mentioned.
 - https://en.wikipedia.org/wiki/String_duality
- So there is a unique theory after all.



T-Duality

- Short for target-space-duality.
- In the simplest example of this relationship, one of the theories describes strings propagating in a spacetime shaped like a circle of some diameter D, while the other theory describes strings propagating on a spacetime shaped like a circle of diameter to 1/D.
 - https://en.wikipedia.org/wiki/T-duality
- For example, suppose you have a calculation that involves D+1/D (for some length, D, say the Planck length, 10⁻³⁵ m) and you get an answer, 10⁻³⁵ m + 10³⁵ m.
- Now do the calculation again for the inverse length, (i.e. D = 10³⁵ m). You get exactly the same answer!

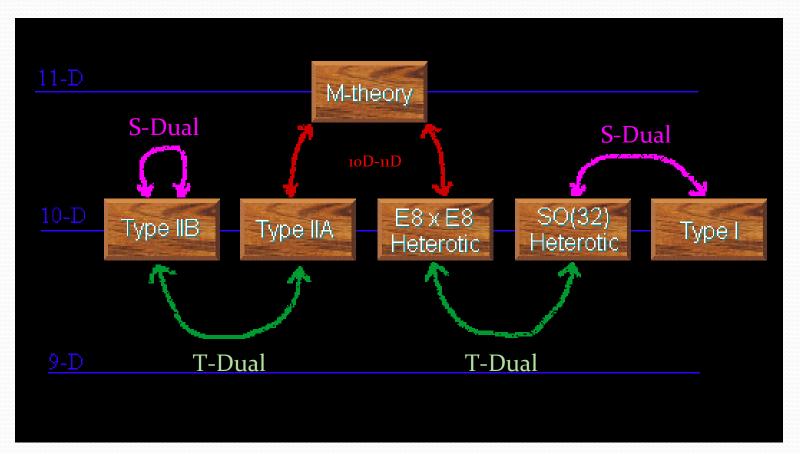


S-Duality

- Short for strong-weak-duality.
- S-duality generalizes classical electrodynamics. In particular, Maxwell's equations give the same result if you interchange the electric and magnetic fields.
 - https://en.wikipedia.org/wiki/S-duality

Dualing Theories

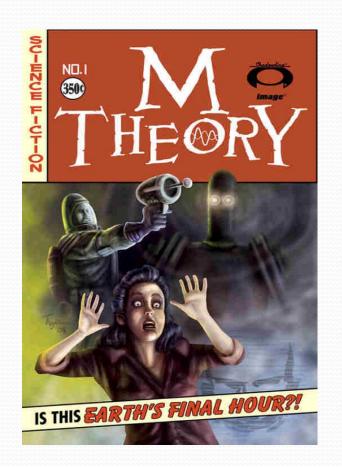
All 5 string theories are duals of each other.



Two of the many scholarly papers on String and M-Theory

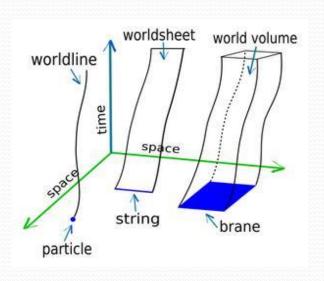
Wayward





Brane New World

- The most important new concept was that membranes (aka branes) were crucial.
 - https://en.wikipedia.org/wiki/D-brane
- A geometric point in the 10-D theory was now extruded into an additional dimension and became a 1-D string.
- An old 1-D string became a 2-D membrane. And so on.



Higher Dimensional Branes

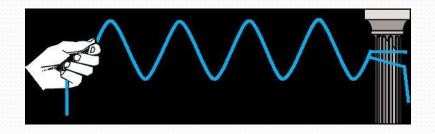
- Branes could come in any number of dimensions up to 10.
- These are called 1-branes, 2-branes, ... 10-branes.
 - In general, these are called *p-branes*.

Brane Sizes

- Branes do not have to be compactified.
- Their size can be large, perhaps even arbitrarily large.
- The remaining dimensions can still be compacted into Calabi-Yau shapes.

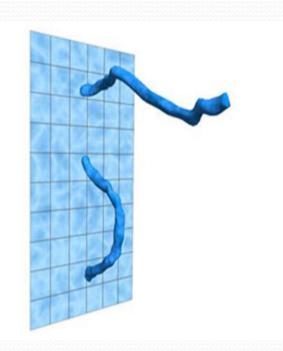
Boundary Conditions

- In general, things that vibrate don't do so infinitely throughout all of time and space.
- Imagine pumping a rope, one end in your hand and the other end tethered to a post.
- They're anchored, often at both ends. These are called boundary conditions.



Why We Can't See Other Branes

- Open strings have a boundary condition. Their ends are restricted to starting and ending on the same brane.
- Closed strings (loops) can't be so restricted.
- Closed strings always represent gravitons.
- All other particles of matter and forces are open strings.
- This might explain why gravity doesn't easily fit in with the Standard Model.



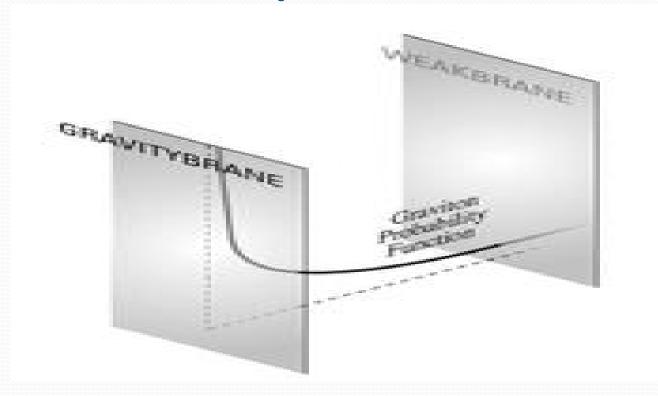
If I Only Had a Brane

- Our entire universe may be a 3-brane embedded in a larger (e.g. 4-) brane.
- With the exception of gravity, all matter and forces could be restricted to our 3-brane.
- So if there were another 3-brane "near" us (in a 4-D sense), we intrinsically couldn't see it.
 - Remember, photons etc. are restricted to our brane.
- And maybe dark matter is matter on another brane!

Branes and the Hierarchy Problem

- The force of gravity is some 10³⁷ times weaker than the electromagnetic force. This is called the *Hierarchy Problem*.
- Lisa Randall has proposed that we're on one brane where gravity is weak but is strong on a nearby brane.
- Gravitons can travel between the branes but weaken by the time they get here.
- The gap between branes is called the bulk.
- The LHC could detect higher dimensions via gravitons leaking into the bulk. This would show up as a violation of the law of conservation of energy.

Gravity in the Bulk



Branes and the Big Bang Theory

- Neil Turok (director emeritus of the Perimeter Institute) and Paul Steinhardt (of Princeton) have proposed the *ekpyrotic* ("out of (cosmic) fire") model to explain the Big Bang. This required two branes to collide and rebound every trillion years or so and thus release energy.
- This avoids a singularity (infinite density) at the Big Bang.
- And it may have left traces in the Cosmic Background Radiation.
- Maybe it's just as well that CBS didn't call their show The Ekpyrotic Model Theory!

Experimental Evidence for Higher Dimensions

The Third Superstring Revolution?

- In 1967, Sir Roger Penrose proposed the concept of 4-dimensional objects called twistors to try to explain how space and time aren't truly fundamental objects.
 - https://en.wikipedia.org/wiki/Twistor_theory
 - http://www.wired.com/science/discoveries/news/2006/09/71828
- In a series of paper in the 90's, Ed Witten incorporated twistors into string theory and proposed a way to do string theory in twistor space, whose dimensionality is necessarily the same as that of traditional 3+1 spacetime.
 - https://en.wikipedia.org/wiki/Twistor-string-theory#:~:text=Twistor%20string-%20theory%20is%20an,by%20Edward%20Witten%20in%202003.
- Hence twistor string theory is a possible way to eliminate the need for more than 3 spatial dimensions when doing string theory.
- However, Witten has said that "I think twistor string theory is something that only partly works." So the jury may still be out on 4-D string theory.

So How Many Dimensions Does the Universe Have?

- Since 10-dimensional string theory is dual to 11dimensional M-Theory, you can assume the universe is 10dimensional and prove your theorems. And as a bonus, you'll have proved a corresponding theorem in an 11dimensional world. And vice-versa.
- And sometimes it's easier to prove a theorem in, say, 11-D than it is to do so in 10-D. And vice-versa.
- On the assumption that M-theory is correct, and does describe our universe, the answer is...
- Either 10 or 11. Take your pick.
- Both ways of looking at it describe identical physics.
 There's no way of telling them apart.

Bottom Line – So Far

- (Super)string theory predicts that we live in a 10/11 dimensional world.
- It can explain a number of aspects of the universe that the Standard Model can't.
- But it's still totally theoretical. At this point it's just "marks on paper".
- All experiments show just 3+1 dimensions and no sign of supersymmetry.
- But there's an awful lot of math and indirect evidence that says that we may be on to something here and it deserves more research.

Some More Recent Work

The Holographic Principle

- There's something called the Holographic Principle that also provides additional reasons why String Theory may be on the right track, but this would take too long to go into.
- Maybe there can be a Holographic Principle presentation at some later date.

Why is the Higgs Boson So Light?

- Very briefly, the Higgs field saturates space, and particles achieve their masses by interacting with the Higgs boson.
- And it works both ways. The Higgs boson gets its mass from all the particles it can interact with, including theoretical super-heavy particles, up to a hundred million billion times the mass of the Higgs.
- So the Higgs boson should be super-heavy itself, which it isn't.
- Why not?

Why is the Higgs Boson So Light?

- Sometimes in particle physics, terms in a calculation cancel.
- For example, suppose a calculation has terms for the momentum of particles, one going left and one going right. The left term would be the negative of the right term and the sum of the two would cancel out.
- For the mass of the Higgs, bosons and fermions contribute positive and negative terms and mostly cancel each other out.
- But in the Standard Model, there are many more types fermions than bosons so it's hard to match things up.
- But with supersymmetry, there are the same number of particles and sparticles and most of the mass calculation terms cancel out.
- See https://www.quantamagazine.org/crisis-in-particle-physics-forces-a-rethink-of-what-is-natural-20220301/

Numerical Evidence???

- Physicists have recently calculated a number associated with quantum gravity by taking known facts and trying to find out what's compatible with them. The actual facts constrain the ultimate theory.
- To their surprise, the answer closely matched the prediction made for this value by string theory.
- https://www.quantamagazine.org/a-correction-to-einstein-hints-at-evidence-for-string-theory-20220121/
- Is this experimental evidence for string theory? No. But pieces of evidence for string theory keep cropping up, which encourages us that maybe we're on the right track.

Unification in General

- The history of science shows many times that things that looked different on the surface were actually the same underneath.
 - Before Newton's theory of gravity, people assumed that objects in the sky operated by different laws than those on earth. For example, the sun rose and set but didn't fall down. But Newton unified celestial and terrestrial gravity.
 - Maxwell's equations unified electricity and magnetism.
 - The 1979 Nobel Prize in Physics was given for unifying electromagnetism and the weak nuclear force.

Unification of Electromagnetism and the Weak Nuclear Force - I

- In the 1960s, the *electroweak theory* was developed by Steven Weinberg, Sheldon Glashow, and Abdus Salam. This theory proposed that the electromagnetic and weak nuclear forces are identical at sufficiently high energies (or equivalently, temperatures). At lower energies, like those in our present-day universe, the two forces remain united but manifest themselves in different ways.
- But at higher energies, experiments show that at extremely short distances and at extremely high energies, the strengths of the forces begin to become more similar.
- https://openstax.org/books/physics/pages/23-3-theunification-of-forces

Unification of Electromagnetism and the Weak Nuclear Force - II

- The force particle (boson) of electromagnetism is the photon, which is massless.
- There are 3 force particles associated with the weak force, namely the W⁺, W⁻ and Z^o. And these are hardly massless, with respective masses of 8o, 8o and 9o GeV.
 - In comparison, the mass of a proton is just under 1 GeV.

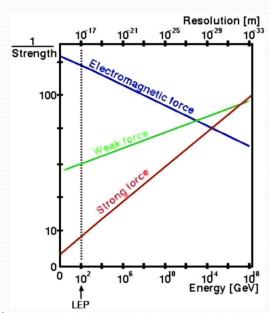
• So how can these two forces be considered the same?

Unification of Electromagnetism and the Weak Nuclear Force - III

- At 10¹⁴ GeV (corresponding to about 10⁻²⁹ m) the strengths of the electromagnetic and weak force are projected to be *almost* the same.
- And while the strong force hasn't been unified with the electroweak force, theory predicts that at the appropriate temperature, all 3 forces will *almost* meet.

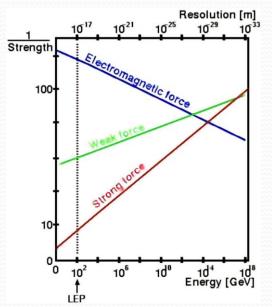
May the Forces Be With Each Other

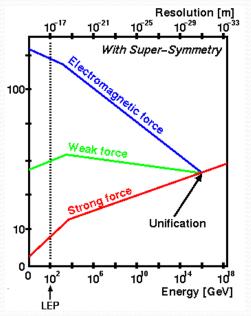
• As we can see in the graph below, the three forces of the Standard Model *almost* meet at about 10¹⁴ GeV.



May the Forces Be With Each Other

- As we can see in the graph below, the three forces of the Standard Model *almost* meet at about 10¹⁴ GeV.
- But if we assume that supersymmetry is valid, then they meet at the same point!





Draw your own conclusions about whether supersymmetry exists...

The Structure of Scientific Revolutions

- Thomas Kuhn, the philosopher of science at MIT, in his book, pointed out that scientists start with a theory of some aspect of the universe (e.g. chemistry, space, time, dark matter, etc.) and incrementally enhance it.
- He called this a paradigm.
- Until it runs into problems. Experimental data that didn't match the paradigm. Things that the current paradigm should be able to explain but didn't. And so on. Like the Standard model.

And so scientists have to come up with a new paradigm.

Bottom Line Redux

- As successful in so many ways as the Standard Model has been, maybe it's time for a new paradigm.
- So maybe additional dimensions and the holographic universe aren't totally off the table.
- Or maybe supersymmetry is true but string theory isn't? I suppose, maybe...
- Do I believe in String Theory? Not necessarily, but things hang together so well and in surprisingly deep ways that maybe the universe is trying to tell us something.

Thank You

- This presentation and part I can be found on my web site in http://www.lrs5.net/FTPData/Science/
- I've given a similar presentation before in 2013. This can be found in the same folder.
- You can also find there two previous NYPG presentations of mine:
 - A Gentle Introduction to Calculus
 - No, E Does Not Equal mc² and I'll Show You Why.
- PDF versions of all these files are also present.